

7.1 动态系统稳定性的基本概念

1. 李雅普诺夫意义下的稳定 (低频振荡、次同步振荡)

$\|x_0 - x_{el}\| < \delta(\varepsilon, t_0) \implies \|\bar{x}(t; x_0, t_0) - x_{el}\| < \varepsilon, t \geq t_0$ x_{el} 为李雅普诺夫意义下的稳定

δ 的取值与 t_0 无关, 则称 x_{el} 是一致稳定

$\varepsilon \downarrow, \delta \downarrow$

2. 渐进稳定

$$\lim_{t \rightarrow \infty} \|\bar{x}(t; x_0, t_0) - x_{el}\| = 0$$

3. 大范围渐进稳定

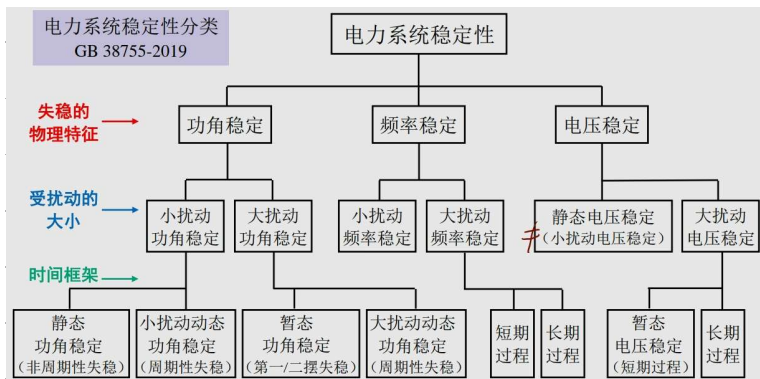
任一非平衡初始状态 x_0 满足 $\lim_{t \rightarrow \infty} \|\bar{x}(t; x_0, t_0) - x_{el}\| = 0$

4. 不稳定

找不到 $\delta(\varepsilon, t_0) > 0$, 使满足 $\|x_0 - x_{el}\| < \delta(\varepsilon, t_0)$ 的任一 x_0 满足 $\|\bar{x}(t; x_0, t_0) - x_{el}\| < \varepsilon$

7.2 电力系统稳定性概述

$$\text{DAE} \begin{cases} \frac{dx}{dt} = f(x, y) \\ 0 = g(x, y) \end{cases} \quad \text{平衡状态} \begin{cases} 0 = f(x_e, y_e) \\ 0 = g(x_e, y_e) \end{cases}$$



1. 功角稳定

静态功角稳定

暂态功角稳定

动态功角稳定

小子扰动态(功角)稳定

大于扰动态(功角)稳定

2. 电压稳定

小子扰动态电压稳定

大扰动电压稳定

短期(暂态)

快速

长期

响应较慢

3. 频率稳定

电力系统稳定性分析方法

小子扰 在平衡点线性化

大于扰 数值积分

7.3 同步发电机组的机电模型

7.3.1 同步发电机组的转子运动方程

$$J\alpha = \Delta M = M_T - M_E \quad \alpha = \frac{d\Omega}{dt} \quad \Omega = \omega/p \quad \frac{d\delta}{dt} = \omega - \omega_0 \quad \frac{d\omega}{dt} = \frac{d^2\delta}{dt^2} = \frac{d^2\delta}{dt^2}$$

$$J \frac{d\Omega}{dt} = J \frac{d\omega}{p dt} = J \frac{d^2\delta}{dt^2} = \Delta M$$

$$\frac{T_J}{\omega_0} \cdot \frac{d\omega}{dt} = \Delta M^* \quad H = T_J/2 \quad \frac{2H}{\omega_0} \cdot \frac{d\omega}{dt} = \Delta M^*$$

ω 变化不大时, $\Delta M^* \approx \Delta P^*$

$$\Downarrow \quad \frac{d\delta}{dt} = (\omega - \omega_0) \quad \frac{T_J}{\omega_0} \cdot \frac{d^2\delta}{dt^2} = P_T - P_E$$

$$\Downarrow \quad \frac{d\omega}{dt} = \frac{1}{T_J} (P_T - P_E)$$

7.3.2 同步发电机的电磁功率

$$M_E = i_q \psi_d - i_d \psi_q$$

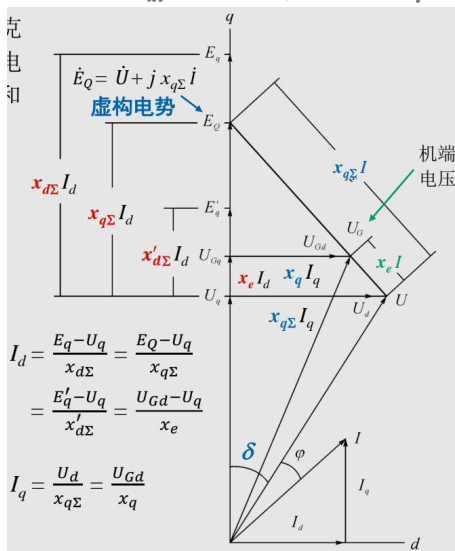
假设

- ① 机组转速接近同步转速，在定子电压方程中认为 $\omega = \omega_0 = 1$; $P_E = M_E \omega = M_E = i_q \psi_d - i_d \psi_q$
- ② 不计定子绕组的电磁暂态过程，即 $\frac{d\psi_d}{dt} = \frac{d\psi_q}{dt} = 0$;
- ③ 忽略定子绕组的电阻，即 $r = 0$;
- ④ 不计阻尼绕组的影响（忽略次暂态，只考虑暂态）。

$$\begin{cases} U_d = \frac{d\psi_d}{dt} - \omega \psi_q - r i_d \\ U_q = \frac{d\psi_q}{dt} + \omega \psi_d - r i_q \end{cases} \Rightarrow \begin{cases} U_d = -\psi_q \\ U_q = \psi_d \end{cases}$$

$$P_E = i_q U_q + i_d U_d$$

发电机

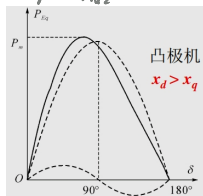


1. E_q 和同步电抗表示的功率表达式

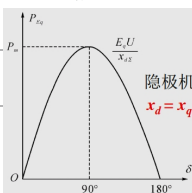
$$U_d = x_q I_q, \quad U_q = E_q - x_d I_d \Rightarrow \begin{cases} I_d = \frac{E_q - U_q}{x_{dE}} \\ I_q = \frac{U_d}{x_{qE}} \end{cases}$$

$$P_{Eg} = \frac{E_q U}{x_{dE}} \sin \delta + \frac{U^2}{2} \frac{x_d - x_q}{x_{dE} x_{qE}} \sin 2\delta$$

$$P_{Eg} = \frac{E_q U}{x_{dE}} \sin \delta$$



小于 90° 处

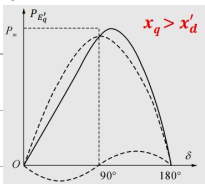


90°处

2. 以 E_q' 和暂态电抗表示的功率表达式

$$\begin{cases} U_d = x_{q\Sigma} I_q \\ U_q = E_q' - x_{d\Sigma}' I_d \end{cases} \Rightarrow \begin{cases} I_d = \frac{E_q' - U_q}{x_{d\Sigma}'} \\ I_q = \frac{U_d}{x_{q\Sigma}} \end{cases}$$

$$P_{E_q'} = \frac{E_q' U}{x_{d\Sigma}'} \sin \delta - \frac{U^2}{2} \frac{x_{q\Sigma} - x_{d\Sigma}'}{x_{d\Sigma}' x_{q\Sigma}} \sin 2\delta$$



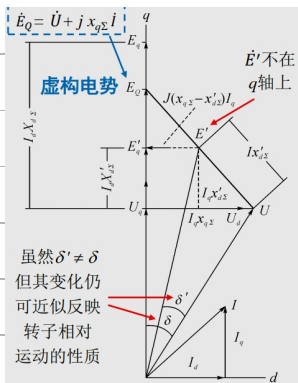
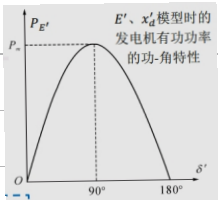
大于90°处

3. 以 E' 表示的功率方程

$$\dot{U} = U_d + jU_q = x_{q\Sigma} I_q + j(E_q' - x_{d\Sigma}' I_d) = \dot{E}' - jx_{d\Sigma}' I$$

暂态电抗后电势 $\dot{E}' = jE_q' - j(x_{q\Sigma} - x_{d\Sigma}') I_q = \dot{E}_q' - j(x_{q\Sigma} - x_{d\Sigma}') I_q \xrightarrow{x_q = x_d'} \dot{E}' = \dot{E}_q'$

$$P_{E'} = \frac{E' U}{x_{d\Sigma}'} \sin \delta'$$



4. 多机电力系统发电机电磁功率的计算

(1) E', x_d' 模型下的发电机电磁功率 (推)

$$\begin{bmatrix} \dot{U}_1 \\ \vdots \\ \dot{U}_n \\ \vdots \\ \dot{E}_1' \\ \vdots \\ \dot{E}_m' \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ I_1 \\ \vdots \\ I_m \end{bmatrix}$$

发电机母线节点
负荷节点
联络节点
电势节点

消去n个零注入节点

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1m} \\ Y_{21} & Y_{22} & \cdots & Y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{m1} & Y_{m2} & \cdots & Y_{mm} \end{bmatrix} \begin{bmatrix} \dot{E}_1' \\ \dot{E}_2' \\ \vdots \\ \dot{E}_m' \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_m \end{bmatrix}$$

$$P_i = \text{Re}(\dot{E}_i' \dot{I}_i) = \text{Re}(\dot{E}_i' \sum_{j=1}^m \dot{E}_j' Y_{ij}) = E_i \sum_{j=1}^m E_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij})$$

$$= E_i^2 G_{ii} + E_i \sum_{j=1, j \neq i}^m E_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij})$$

$$P_i = E_i^2 G_{ii} + E_i \sum_{j=1, j \neq i}^m E_j |Y_{ij}| \sin(\delta_{ij} + \beta_{ij})$$

12) $E'_q, x'_d(x_q)$ 模型下的发电机电磁功率 凸

$$P_E = U_{Gx} I_{Gx} + U_{Gy} I_{Gy}$$

$$\begin{bmatrix} U_d \\ U_q \end{bmatrix} = \begin{bmatrix} 0 \\ E'_q \end{bmatrix} - \begin{bmatrix} 0 & -x_q \\ x'_d & 0 \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix}$$

$$\begin{bmatrix} U_d \\ U_q \end{bmatrix} = \begin{bmatrix} \sin \delta & -\cos \delta \\ \cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} U_{Gx} \\ U_{Gy} \end{bmatrix}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \sin \delta & -\cos \delta \\ \cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} I_{Gx} \\ I_{Gy} \end{bmatrix}$$

$$\begin{bmatrix} \sin \delta & -\cos \delta \\ \cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} U_{Gx} \\ U_{Gy} \end{bmatrix} = \begin{bmatrix} 0 \\ E'_q \end{bmatrix} - \begin{bmatrix} 0 & -x_q \\ x'_d & 0 \end{bmatrix} \begin{bmatrix} \sin \delta & -\cos \delta \\ \cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} I_{Gx} \\ I_{Gy} \end{bmatrix}$$



$$\begin{bmatrix} I_{Gx} \\ I_{Gy} \end{bmatrix} = \begin{bmatrix} G_x & B_x \\ B_y & G_y \end{bmatrix} \begin{bmatrix} E'_q \cos \delta \\ E'_q \sin \delta \end{bmatrix} - \begin{bmatrix} G_x & B_x \\ B_y & G_y \end{bmatrix} \begin{bmatrix} U_{Gx} \\ U_{Gy} \end{bmatrix} = \begin{bmatrix} I'_{Gx} \\ I'_{Gy} \end{bmatrix} - \begin{bmatrix} G_x & B_x \\ B_y & G_y \end{bmatrix} \begin{bmatrix} U_{Gx} \\ U_{Gy} \end{bmatrix}$$

$$\begin{aligned} G_x &= [(x_q - x'_d) \sin 2\delta] / (2x'_d x_q) & B_x &= [(x_q + x'_d) - (x_q - x'_d) \cos 2\delta] / (2x'_d x_q) \\ G_y &= -G_x & B_y &= -[(x_q + x'_d) + (x_q - x'_d) \cos 2\delta] / (2x'_d x_q) \end{aligned}$$



$$\begin{aligned} \begin{bmatrix} I_{Gx} \\ I_{Gy} \end{bmatrix} &= \begin{bmatrix} G_x & B_x \\ B_y & G_y \end{bmatrix} \begin{bmatrix} E'_q \cos \delta \\ E'_q \sin \delta \end{bmatrix} - \begin{bmatrix} G_x & B_x \\ B_y & G_y \end{bmatrix} \begin{bmatrix} U_{Gx} \\ U_{Gy} \end{bmatrix} = \begin{bmatrix} I'_{Gx} \\ I'_{Gy} \end{bmatrix} - \begin{bmatrix} G_x & B_x \\ B_y & G_y \end{bmatrix} \begin{bmatrix} U_{Gx} \\ U_{Gy} \end{bmatrix} \\ &= \begin{bmatrix} G_x & B_x \\ B_y & G_y \end{bmatrix} \begin{bmatrix} E'_q \cos \delta \\ E'_q \sin \delta \end{bmatrix} - \begin{bmatrix} G_x & B_x \\ B_y & G_y \end{bmatrix} \begin{bmatrix} U_{Gx} \\ U_{Gy} \end{bmatrix} = \begin{bmatrix} I'_{Gx} \\ I'_{Gy} \end{bmatrix} - \begin{bmatrix} G_x & B_x \\ B_y & G_y \end{bmatrix} \begin{bmatrix} U_{Gx} \\ U_{Gy} \end{bmatrix} \\ &\text{与 } \delta \text{ 无关} \quad \text{但与 } U_G \text{ 有关} \end{aligned}$$

常数矩阵

$$\begin{aligned} B'_x &= (x_q + x'_d) / (2x'_d x_q) & B'_y &= -B'_x \\ B''_x &= -(x_q - x'_d) \cos 2\delta / (2x'_d x_q) & B''_y &= B''_x \end{aligned}$$

7.4 电力系统静态稳定

7.4.1 静态稳定分析的基本方法

1. 简单系统静态稳定性分析的同步功率系数法

静态稳定的实用判据 $dP_E / d\delta > 0$

$$\text{同步功率系数 } \frac{dP_E}{d\delta} = \frac{E_q U}{x_{dz}} \cos \delta \text{ 越大, 稳定性 } \uparrow$$

$$\text{储备系数 } k_p = \frac{P_{max} - P_0}{P_0}$$

正常运行 15% ~ 20%
事故后 10%

2. 小干扰法

非线性微分方程组 $\rightarrow$ 线性化

$$\text{阻尼比 } \zeta_k = \frac{-\sigma_k}{\sqrt{\sigma_k^2 + \omega_k^2}}$$

7.4.2 简单电力系统的静态稳定分析

1. 系统的状态方程

$$\textcircled{1} \frac{d\delta}{dt} = (\omega - 1)\omega_0$$

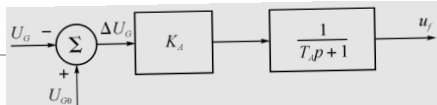
$$\textcircled{2} \frac{d\omega}{dt} = \frac{1}{T_J} [P_T - P_E - D(\omega - 1)] \quad P_E = P_{E_q}' = \frac{E_q' U}{x_{d\Sigma}'} \sin \delta - \frac{U^2}{2} \frac{x_q - x_d'}{x_{d\Sigma}' x_{q\Sigma}'} \sin 2\delta$$

$$\textcircled{3} T_{d0}' \frac{dE_q'}{dt} = E_{qe} - E_q \quad \leftarrow \text{由励磁绕组电压方程式}$$

$$\frac{x_f}{r_f} \quad \frac{x_{ad}}{x_f} \quad \frac{x_{ad}}{x_f} \quad \frac{U_f}{r_f} \quad x_{ad} \frac{U_f}{r_f}$$

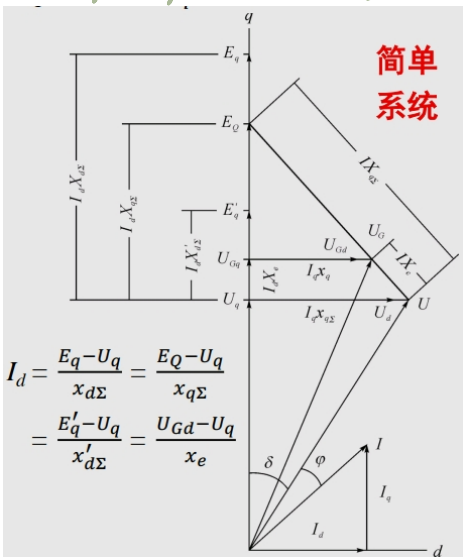
强励空载电压

比例型励磁调节系统



$$\textcircled{4} T_A \frac{dE_{qe}}{dt} = -E_{qe} + K_A (U_{G0} - U_G)$$

2. ΔP_{E_q} 、 ΔE_q 、 ΔU_G 的 6 个线性化系数



$$\Delta P_{E_q}' = \frac{\partial P_{E_q}'}{\partial \delta} \Delta \delta + \frac{\partial P_{E_q}'}{\partial E_q'} \Delta E_q' = K_1 \Delta \delta + K_2 \Delta E_q'$$

$$K_1 = S_{E_q'} \triangleq \frac{\partial P_{E_q}'}{\partial \delta} = \frac{E_q' U}{x_{d\Sigma}'} \cos \delta - U^2 \frac{x_q - x_d'}{x_{d\Sigma}' x_{q\Sigma}'} \cos 2\delta, \quad K_2 \triangleq \frac{\partial P_{E_q}'}{\partial E_q'} = \frac{U}{x_{d\Sigma}'} \sin \delta$$

同步功率系数

由于 $E_q = U_q + x_{d\Sigma} I_d$, $E'_q = U_q + x'_{d\Sigma} I_d$,

故有 $E_q = \frac{x_{d\Sigma}}{x'_{d\Sigma}} E'_q - \frac{x_d - x'_d}{x'_{d\Sigma}} U \cos \delta$, 从而

$$\Delta E_q = \frac{\partial E_q}{\partial E'_q} \Delta E'_q + \frac{\partial E_q}{\partial \delta} \Delta \delta = \frac{1}{K_3} \Delta E'_q + K_4 \Delta \delta$$

$$\frac{1}{K_3} \triangleq \frac{\partial E_q}{\partial E'_q} = \frac{x_{d\Sigma}}{x'_{d\Sigma}}, \quad K_4 \triangleq \frac{\partial E_q}{\partial \delta} = \frac{x_d - x'_d}{x'_{d\Sigma}} U \sin \delta$$

由于 $U_{Gd} = x_q I_q = x_q \frac{U_d}{x_{q\Sigma}} = \frac{x_q}{x_{q\Sigma}} U \sin \delta$,

$$U_{Gq} = U_q + (x_{d\Sigma} - x_d) I_d = U_q + (x_{d\Sigma} - x_d) \frac{E'_q - U_q}{x'_{d\Sigma}} = \frac{x'_d}{x'_{d\Sigma}} U \cos \delta + \frac{x_{d\Sigma} - x_d}{x'_{d\Sigma}} E'_q$$

而 $U_G^2 = U_{Gd}^2 + U_{Gq}^2$, 故 $\Delta U_G = \frac{\partial U_G}{\partial \delta} \Delta \delta + \frac{\partial U_G}{\partial E'_q} \Delta E'_q = K_5 \Delta \delta + K_6 \Delta E'_q$

$$K_5 \triangleq \frac{\partial U_G}{\partial \delta} = \frac{U_{Gd}}{U_G} \frac{x_q}{x_{q\Sigma}} U \cos \delta - \frac{U_{Gq}}{U_G} \frac{x'_d}{x'_{d\Sigma}} U \sin \delta, \quad K_6 \triangleq \frac{\partial U_G}{\partial E'_q} = \frac{U_{Gq}}{U_G} \frac{x_{d\Sigma} - x_d}{x'_{d\Sigma}}$$

3. 静态稳定性分析

(1) 不考虑自动励磁调节器 U_f (或 E_{Gf}) 恒定, $\frac{dE_{Gf}}{dt} = 0$ 去④ $\begin{cases} \frac{d\delta}{dt} = (\omega - 1)\omega_0 \\ \frac{d\omega}{dt} = \frac{1}{T_J} [P_T - P_{E'_q} - D(\omega - 1)] \end{cases}$

① 不计励磁绕组的电磁暂态过程 E'_q 恒定, $\frac{dE'_q}{dt} = 0$ 去③

a) 不计发电机的阻尼作用 (二阶) $D = 0$

$\Delta E'_q = 0$, 故 $\Delta P_{E'_q} = K_1 \Delta \delta + K_2 \Delta E'_q = K_1 \Delta \delta$

二阶系统的增量方程
$$\begin{bmatrix} \frac{d\Delta\delta}{dt} \\ \frac{d\Delta\omega}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 \\ -\frac{K_1}{T_J} & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} = A \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix}$$

特征根 $\lambda_{1,2} = \pm \sqrt{-\frac{\omega_0 K_1}{T_J}} = \pm j\omega \quad \omega = \sqrt{\frac{\omega_0 K_1}{T_J}}$ 自由振荡角频率

$\begin{cases} K_1 > 0 & \text{等幅振荡} \\ K_1 < 0 & \text{存在正根, 非周期性失去稳定} \end{cases}$

b) 计及发电机的阻尼作用 (二阶) $D \neq 0$

二阶系统的增量方程
$$\begin{bmatrix} \frac{d\Delta\delta}{dt} \\ \frac{d\Delta\omega}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 \\ -\frac{K_1}{T_J} & -\frac{D}{T_J} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} = A \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix}$$

特征根 $\lambda_{12} = -\frac{D}{2T_J} \pm \frac{1}{2T_J} \sqrt{D^2 - 4\omega_0 T_J K_1}$

静态稳定判据 $D > 0, K_1 > 0$

② 计及励磁绕组的电磁暂态过程(三阶)

$$\begin{cases} \frac{d\delta}{dt} = (\omega - 1)\omega_0 \\ \frac{d\omega}{dt} = \frac{1}{T_J} (P_T - P_{E'_q}) \\ T'_{d0} \frac{dE'_q}{dt} = E_{qe} - E_q \end{cases}$$

(2) 考虑自动励磁调节器

$$\begin{cases} \frac{d\delta}{dt} = (\omega - 1)\omega_0 \\ \frac{d\omega}{dt} = \frac{1}{T_J} (P_T - P_{E'_q}) \\ T'_{d0} \frac{dE'_q}{dt} = E_{qe} - E_q \\ T_A \frac{dE_{qe}}{dt} = -E_{qe} + K_A (U_{G0} - U_G) \end{cases}$$

① $T_A = 0$

增量方程

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{E}'_q \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 \\ -\frac{K_1}{T_J} & 0 & -\frac{K_2}{T_J} \\ -\frac{1}{T'_{d0}}(K_4 + K_A K_5) & 0 & -\frac{1}{T'_{d0}}\left(\frac{1}{K_3} + K_A K_6\right) \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \end{bmatrix}$$

稳定判据

$$\begin{aligned} a_1 &= \frac{1 + K_3 K_A K_6}{T'_{d0} K_3} > 0 & a_3 &= \frac{\omega_0}{T'_{d0} T_J} \left[\left(\frac{K_1}{K_3} - K_2 K_4 \right) + K_A (K_1 K_6 - K_2 K_5) \right] > 0 \\ a_2 &= \frac{\omega_0 K_1}{T_J} > 0 & \Delta_2 &= a_1 a_2 - a_0 a_3 = \frac{\omega_0 K_2}{T'_{d0} T_J} (K_4 + K_A K_5) > 0 \end{aligned}$$

$$\begin{cases} K_A, K_3, K_6 > 0, \delta_0 \in (0^\circ, 180^\circ) \text{ 时 } K_2, K_4 > 0 \\ K_5 < 0 \\ K_1 = \frac{\partial P_{E'_q}}{\partial \delta} \end{cases}$$

静态稳定判据

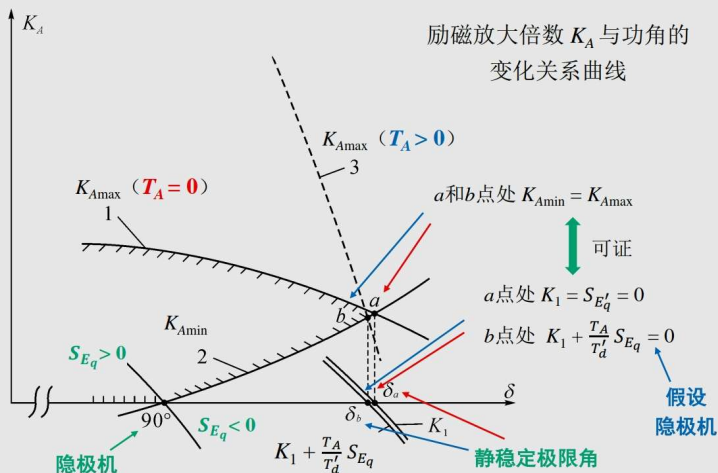
$$K_1 > 0$$

$$K_A > -\frac{K_1 - K_2 K_4 K_6}{K_3 (K_1 K_6 - K_2 K_5)} = K_{Amin} \quad (\text{励磁调节器放大倍数的下限值})$$

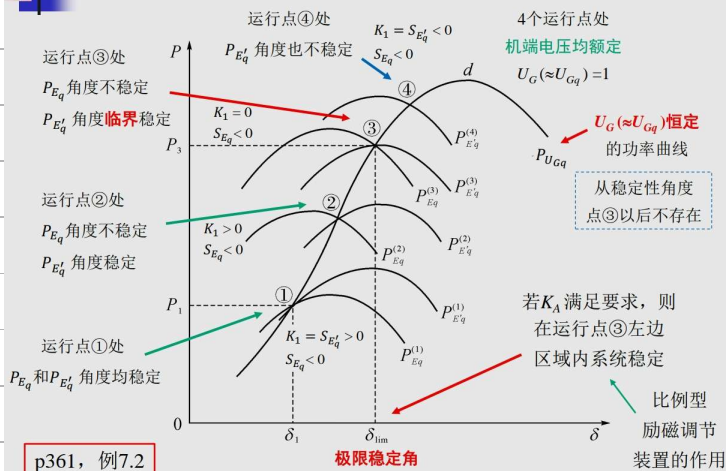
$$K_{Amin} \leq K_A \leq K_{Amax}$$

$$K_A < -\frac{K_1}{K_5} = K_{Amax} \quad (\text{励磁调节器放大倍数的上限值})$$

励磁放大倍数 K_A 与功角的关系



简单系统的功率曲线及系统的静态稳定区



p361, 例7.2

$$② T_A > 0$$

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \\ \Delta \dot{E}'_q \\ \Delta \dot{E}_{qe} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ -\frac{K_1}{T_j} & 0 & -\frac{K_2}{T_j} & 0 \\ -\frac{K_4}{T'_{d0}} & 0 & -\frac{1}{K_3 T'_{d0}} & \frac{1}{T'_{d0}} \\ -\frac{K_A K_5}{T_A} & 0 & -\frac{K_A K_6}{T_A} & -\frac{1}{T_A} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E_{qe} \end{bmatrix} = A \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E'_q \\ \Delta E_{qe} \end{bmatrix}$$

隐极机时的稳定条件

$$K_1 + \frac{T_A}{T_d'} S_{E_q} > 0 \text{ 且 } K_{Amin} \leq K_A \leq K_{Amax}$$

7.4.3 多机电力系统的静态稳定分析

采用 E' 恒定, 电抗为 X_{ij} 的 2 阶模型

不计阻尼时, 发电机的运动方程

$$\begin{aligned} \frac{d\delta_i}{dt} &= (\omega_i - 1)\omega_0 \\ \frac{d\omega_i}{dt} &= \frac{1}{T_{ji}} (P_{Ti} - P_{Ei}) \end{aligned} \quad \rightarrow \quad \frac{d^2\delta_i}{dt^2} = \frac{1}{T_{ji}} (P_{Ti} - P_{Ei})$$

多机电力系统的 P_{Ei}

$$P_{Ei} = E_i^2 G_{ii} + \sum_{j=1, j \neq i}^n E_j |Y_{ij}| \sin(\delta_j + \beta_{ij})$$

这里的 E 实为 E'

↓ 全微分

$$\Delta P_{Ei} = \frac{\partial P_{Ei}}{\partial \delta_i} \Delta \delta_i + \sum_{j=1, j \neq i}^n \frac{\partial P_{Ei}}{\partial \delta_j} \Delta \delta_j = K_{ii} \Delta \delta_i + \sum_{j=1, j \neq i}^n K_{ij} \Delta \delta_j$$

其中, $K_{ii} \triangleq \frac{\partial P_{Ei}}{\partial \delta_i}$, $K_{ij} \triangleq \frac{\partial P_{Ei}}{\partial \delta_j}$, 也称 **同步功率系数**。

7.5 电力系统暂态稳定

大扰动 一短路故障最严重

假设:

- 忽略发电机定子绕组和电力网中**电磁暂态过程**的影响, 只考虑交流系统中**基波**分量电压、电流和功率以及发电机**转子**绕组中**非周期**分量的变化。这样交流电力网中各元件的数学模型可以简单地用它们的**基波阻抗**电路来描述, 电力网络可用**代数方程**表述。
- 不对称**故障或非全相运行期间, 略去发电机定子回路基波**负序**分量电压、电流对电磁转矩的影响。至于基波**零序**分量电流, 由于一般不能流过定子绕组, 故无需考虑。因此, 在发生不对称故障时, 电力网可用**正序增广网络**表示。

数学模型简化 ① 发电机采用 E'_q 恒定或 E' 恒定

② 原动机输入机械功率恒定

③ 负荷以恒定阻抗表示

7.5.1 简单系统暂态稳定性分析的等面积定则

1. 物理分析过程

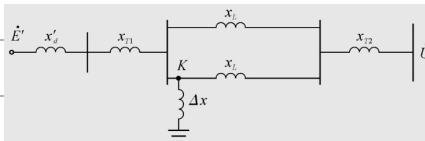
正常运行时



$$x_1 = x_d' + x_{T1} + x_L/2 + x_{T2},$$

$$P_1 = \frac{E'U}{x_1} \sin \delta = P_{1m} \sin \delta$$

故障中



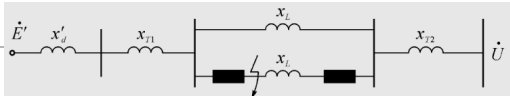
$$x_{II} = x_d' + x_{T1} + x_L/2 + x_{T2} + \frac{(x_d' + x_{T1})(x_L/2 + x_{T2})}{\Delta x}$$

$$P_{II} = \frac{E'U}{x_{II}} \sin \delta = P_{IIm} \sin \delta$$

金属性三相短路, $\Delta x = 0, x_{II} = \infty,$

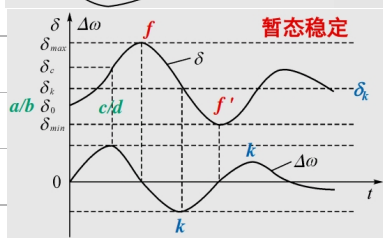
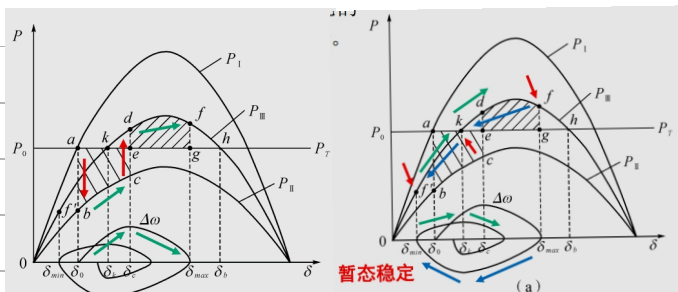
$$P_{II} = 0$$

故障后



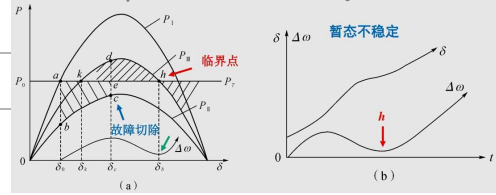
$$x_{III} = x_d' + x_{T1} + x_L + x_{T2}$$

$$P_{III} = \frac{E'U}{x_{III}} \sin \delta = P_{III m} \sin \delta$$

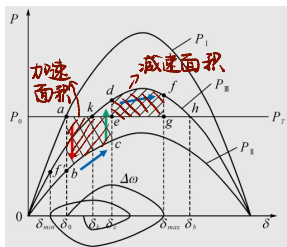


暂态稳定

切除故障时间过长 $\rightarrow$ 失步



2. 等面积定则



加 $S =$ 减 S

最大减速面积 $edfhe$

$\delta_c \downarrow$, 最大减 $S \uparrow$

极限切除角

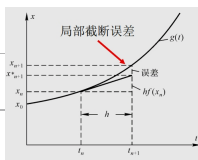
$$\delta_{cm} = \arccos \frac{P_T(\delta_h - \delta_0) + P_{lim} \cos \delta_h - P_{lim} \cos \delta_0}{P_{lim} - P_{lim}}$$

7.5.2 暂态稳定性分析的数值积分方法

1. 显式积分算法

(1) 欧拉法

$$\frac{dx}{dt} = f(x) \quad x_{n+1} = x_n + hf(x_n)$$



$$\text{局部截断误差 } E_{n+1} \leq \frac{M}{2} h^2$$

全局截断误差与 h 成正比
但小步长引起舍入误差

(2) 改进欧拉法

$$x_{n+1} = x_n + h \cdot \frac{f(x_n) + f(x_{n+1})}{2}$$

局部截断误差 $\propto h^3$, 全局截断误差 $\propto h^2$

(1) 简单系统暂态稳定计算的欧拉法

$$\begin{aligned} \frac{d\omega}{dt} &= \frac{\omega_0}{T_J} (P_T - P_m \sin \delta) = \alpha & \rightarrow & \omega_{n+1} = \omega_n + \alpha_n \Delta t & \rightarrow & \Delta \omega_{n+1} = \Delta \omega_n + \alpha_n \Delta t \\ \frac{d\delta}{dt} &= \omega - 1 = \Delta \omega & \rightarrow & \delta_{n+1} = \delta_n + \Delta \omega_n \Delta t & \rightarrow & \Delta \delta_{n+1} = \Delta \omega_n \Delta t \end{aligned}$$

2) 简单系统暂态稳定计算的改进欧拉法

2. 隐式积分算法

$$\frac{dx}{dt} = f(x) \quad x_{n+1} = x_n + \frac{h}{2} [f(x_n) + f(x_{n+1})]$$

3. 微分-代数方程组的数值解法

$$\begin{cases} \frac{dx}{dt} = f(x, y) \\ 0 = g(x, y) \end{cases} \xrightarrow{\text{隐式梯形积分}} \begin{cases} x_{n+1} = x_n + \frac{h}{2} [f(x_n, y_n) + f(x_{n+1}, y_{n+1})] \\ 0 = g(x_{n+1}, y_{n+1}) \end{cases}$$

联立求解法 无交接误差
交替求解法 有 . . .

4. 数值积分法暂态稳定计算的基本流程

7.5.3 暂态稳定分析的李雅普诺夫直接法概述

1. 直接法的概念

临界能量 V_{cr} $V > V_{cr}$ 不稳定

稳定裕度 $V_{cr} - V$

特点: 直接判别稳定性

2. 简单系统的直接法暂稳分析

动能 $V_k^c = \text{加速面积 } S_A$

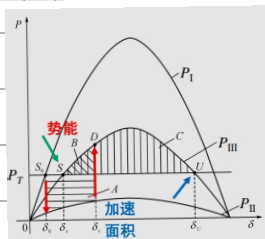
$$\Rightarrow V^c = V_k^c + V_p^c = S_A + S_B = S_{(A+B)}$$

势能 $V_p^c = S_B$

最大减速面积 S_C

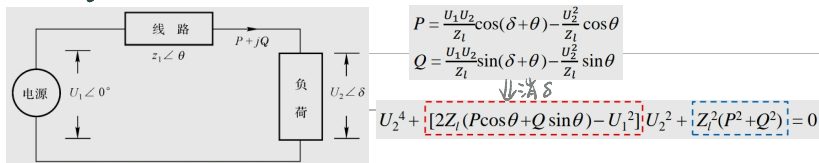
临界能量 $V_{cr} = S_B + S_C$

规格化的稳定度 $\Delta V_n = \frac{V_{cr} - V^c}{V_k^c}$



7.6 提高电力系统稳定性的措施

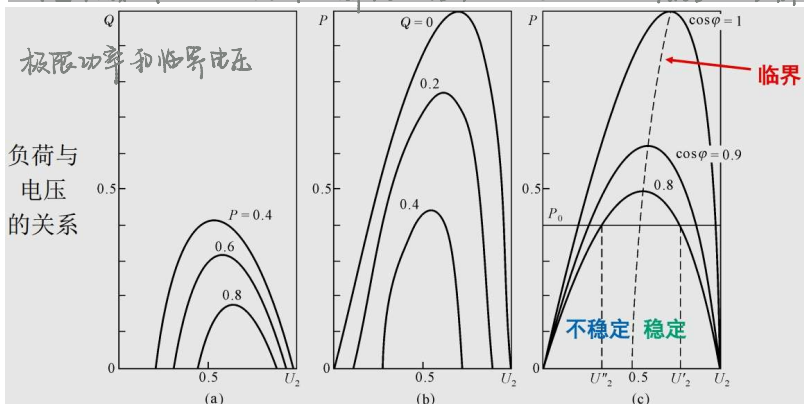
7.7 电力系统的电压稳定



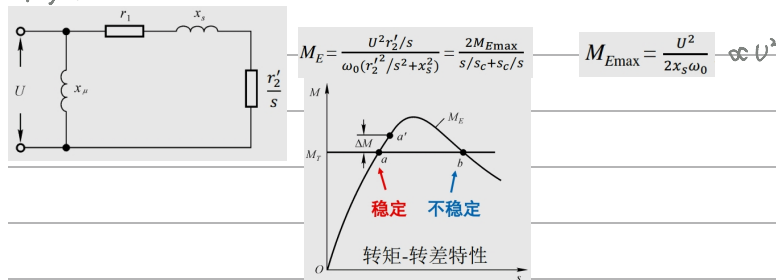
同一负荷下, U_2 有两解, 大的 U_2 静态稳定, 小的 U_2 不稳定。

负荷 ↑, U_2 ↓, U_2 ↑ → 一个解(临界) → 无解(电压崩溃)

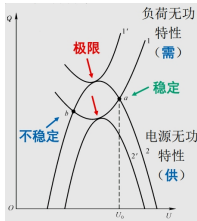
增至 $P_{\max}$ 或 $Q_{\max}$ 时, U_2 仅一解, $U_2 = U_2''$, $dP/dU_2 = 0$ or $dQ/dU_2 = 0 \Rightarrow$ 新稳判据



异步电动机运行点的小干扰稳定性



电压稳定与无功功率的供需平衡



稳定运行条件

$$\frac{d\Delta Q}{dU} = \frac{d(Q_1 - Q_2)}{dU} < 0$$

极限情况

$$\frac{d\Delta Q}{dU} = 0$$